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NAVAL POSTGRADUATE SCHOOL

Monterey, California

AERODYNAMIC STABILIZATION OF GASEOUS DISCHARGES

O. Biblarz

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NAVAL POSTGRADUATE SCHOOL

Monterey, California

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This report reflects work performed during AY 1976-77 while the author was on temporary duty at the Department of Aeronautics at the Technion in Haifa, Israel.

The author would like to thank his colleagues and students at the Technion for helping make his stay a memorable experience.

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This report covers some aspects of the formulation of the stabilization problem by aerodynamic means. Results of experimental work are not included but are referred to in appropriate places. The classical solution for ambipolar diffusion in the positive column of the glow discharge must be extended to reflect the influence of current and pressure. The influence of current is given by the inclusion of appropriate energy equations and the pressure influence by the addition of two-body recombination. It was found that, from dimensional analysis, the characteristic lengths for ambipolar and heat diffusion for

helium do not match. A re-formulation of the problem is thus necessary. The recombination term does not change the eigenvalue nature of the diffussion equation except at the stationary or local equilibrium limit.

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I. INTRODUCTION

In the high energy laser field, the electric discharge convection laser (EDCL) offers good efficiency as well as simplicity of handling relative to gasdynamic and chemical lasers. Progress towards higher laser outputs from EDCLs, however, has been impeded by the collapse of the glow discharge at the increased currents and pressures desired. This glow collapse (usually referred to as arcing or constriction) represents the practical upper limit of the electrical power that may be pumped into the necessary non-equilibrium processes. It has been shown[1-4] that aerodynamic stabilization can be used very successfully in overcoming the glow-collapse limits. It should be pointed out here that the laser application is but one application of the relatively new interdisciplinary field of cold plasma chemistry.

Aerodynamic stabilization is based on two separate effects usually referred to as convection (laminar) and mixing. The purely convective effect is said to decrease the residence time of the particles through the discharge and thereby to counteract unstable mechanism that lead to the glow collapse[5]. The mixing effect, which may be due to vortex flows or to turbulence from boundary layers or grids, has been observed to have powerful stabilizing characteristics. Convection scales up with flow velocity whereas mixing depends on the intensity, spectral distribution, and location of the eddies or vortices.

II. SPECIFICS

At the Technion, the author had the opportunity to participate in experimental work dealing with vortex-flow stabilization[6,7] and with supersonic flow stabilization[6,8]. These techniques complement the work that has been underway at the Postgraduate School on turbulence stabilization of discharges. Some results of the Technion work have already appeared (see Refs. 6-11) and more will be forthcoming.

A significant portion of the author's time was dedicated to a study of theoretical aspects of discharge stabilization. The author was privileged to colaborate with Dr. Y. Khait in establishing a perspective regarding some of the author's previous results with turbulence stabilization. This effort resulted in the writing of a paper titled "Influence of Turbulence on Diffuse Electrical Gas Discharges Under Moderate Pressures," which is being submitted for publication. Other theoretical work involved a close look at the Schottky solution for the positive column in a glow discharge with the aim at establishing its use for a stability formulation. This effort was fruitful on two intermediate aspects of the problem and these are included here as Appendixes A and B.

A proposal was written to the United States-Israel Binational Science Foundation[10] which extensively outlines the progress of the entire effort. If awarded, the cooperation of the Technion and the Postgraduate School would be formalized (however, the budget would provide neither salaries for principal investigators nor equipment for the USA experiments).

III. DISCUSSION

The classical solution to the ambipolar diffusion of charges in the positive column[12,13] of a glow discharge offers no hints as to how striations and constriction, among other important observations, manifest themselves in the discharge. As pressure and/or current are increased, instabilities grow in the discharge and it is important to have a sufficiently complete model of the physics which reflects these effects. In other words, before flow effects such as (laminar) convection and turbulence are introduced it is necessary to re-examine some assumptions inherent in the Schottky solution.

A partial list of phenomena not reflected in the Schottky solution may be given as follows:

1) effect of sheaths at the walls
2) effect of electrodes and intermediate regions
3) effect of recombination in the gas (pressure and composition)
4) effects of the energy balance, including radiation and the elevation of the electron temperature

The list can be rather long. We closely examined the energy balance because the instabilities are believed to be thermal disturbances (i.e., the temperature becomes non-homogeneous) and recombination in the gas because it closely reflects the effect of pressure and composition. For simplicity, a two-dimensional (infinite parallel plate) geometry was considered, see Fig. 1.

When E_L is constant, $\nabla \cdot \bar{J} = 0$ implies that $J_x = J(y)$ or that $n = n(y)$ because $J_y = 0$; moreover, $\nabla \times \bar{E} = 0$ implies that $E_y = E(y)$. The quasineutral equations for steady state may be written as[10,14]

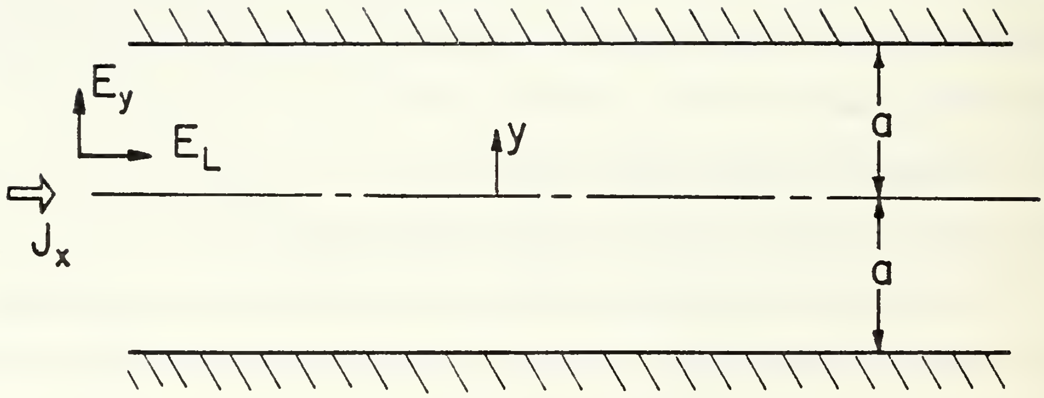


Figure 1. Geometry of Positive Column

$$- \partial/\partial y \left[\frac{D_a}{2T} \frac{\partial}{\partial y} (nT_e)\right] = \nu_i n - \alpha n^2 \tag{1}$$

$$- \partial/\partial y \, [K \, \partial T/\partial y] = \bar{J}\cdot\bar{E} = en\mu_e E_L^2 \tag{2}$$

$$2 \frac{me}{m_N} \nu_{eN} \frac{3}{2} k(T_e - T) = e\mu_e E_L^2 \tag{3}$$

$$p = NkT \tag{4}$$

with boundary conditions

at $y = 0$ $\quad \partial n/\partial y = \partial T/\partial y = 0$

at $y = \pm a$ $\quad n \cong 0$ and $T = T_o$

In the above formulation x is the axial coordinate and y the interplate coordinate, n the charge concentration in the ambipolar region, T_e the electron temperature and T the gas/ion temperature, N is the concentration of the neutrals, D_a the coefficient of ambipolar diffusion, K the thermal conductivity, m_e the electron mobility, ν_i and α the ionization and recombination coefficients respectively, E_L the axial electric field, ν_{eN} the collision frequency between electrons and neutrals, e the electronic charge, p the pressure, and m_e and m_N the mass of the electrons and neutral respectively. There are several assumptions inherent in this formulation, namely that $N>>n$, $T_e>T$, and that p = constant. In addition, we are using a symmetry condition in the boundary conditions and we are neglecting of wall space charges. Glow discharge data[12,13], in addition, indicate that E_L as well as T_e appear to be constant in the discharge at least before constricting. The electron energy equation, Eqn. 3, should have to reflect the energy dependance of the collision cross sections and, in addition, non-elastic collisions -- since we are interested in laser operation. However, we shall not deal with Eqns. (3) and (4).

Using simple kinetic theory to describe the gas temperature dependance of the coefficients (D_a , μ_e , K , etc.) and defining a stationary state where

$$\nu_i n^* - \alpha n^{*2} = 0 \tag{5}$$

$$\text{or} \qquad n^* = \nu_i/\alpha$$

we can proceed to nondimensionalize Eqns. (1) and (2). We shall, however, not use the plate spacing to render y dimensionless, but introduce two characteristic lengths, L_n and L_T[15]. The gas temperature is made non-dimensional by the use of a wall temperature T_o . The resulting equations are:

$$- d/dy \left(T^{1/2} \frac{dn}{dy}\right) = n - n^2 \tag{6}$$

$$L_n \equiv \sqrt{\frac{T_e D_{ao}}{2\nu_{io} T_o}} \tag{7}$$

$$- d/dy \left(T^{3/2} \frac{dT}{dy}\right) = nT \tag{8}$$

$$L_T \equiv \sqrt{\frac{T_o K_o}{en^* \mu_{eo} E_L^2}} \tag{9}$$

It seems reasonable to expect that in a conventional (unconstricted) glow, the two characteristic lengths be about equal to each other and that

$$L_n/a \approx O(1) \quad \text{and} \quad L_T/a \approx O(1) \tag{10}$$

This expectation is also required by the eigenvalue nature of Eqns. (6) and (8). It may be shown (see Appendix B), that except for the equilibrium limit, Eqn. (6) is an eigenvalue equation, (the classical solution being a cosine when no recombination is present and when $T \approx$ constant). The nature of Eqn. (6) is discussed further at the end of this section.

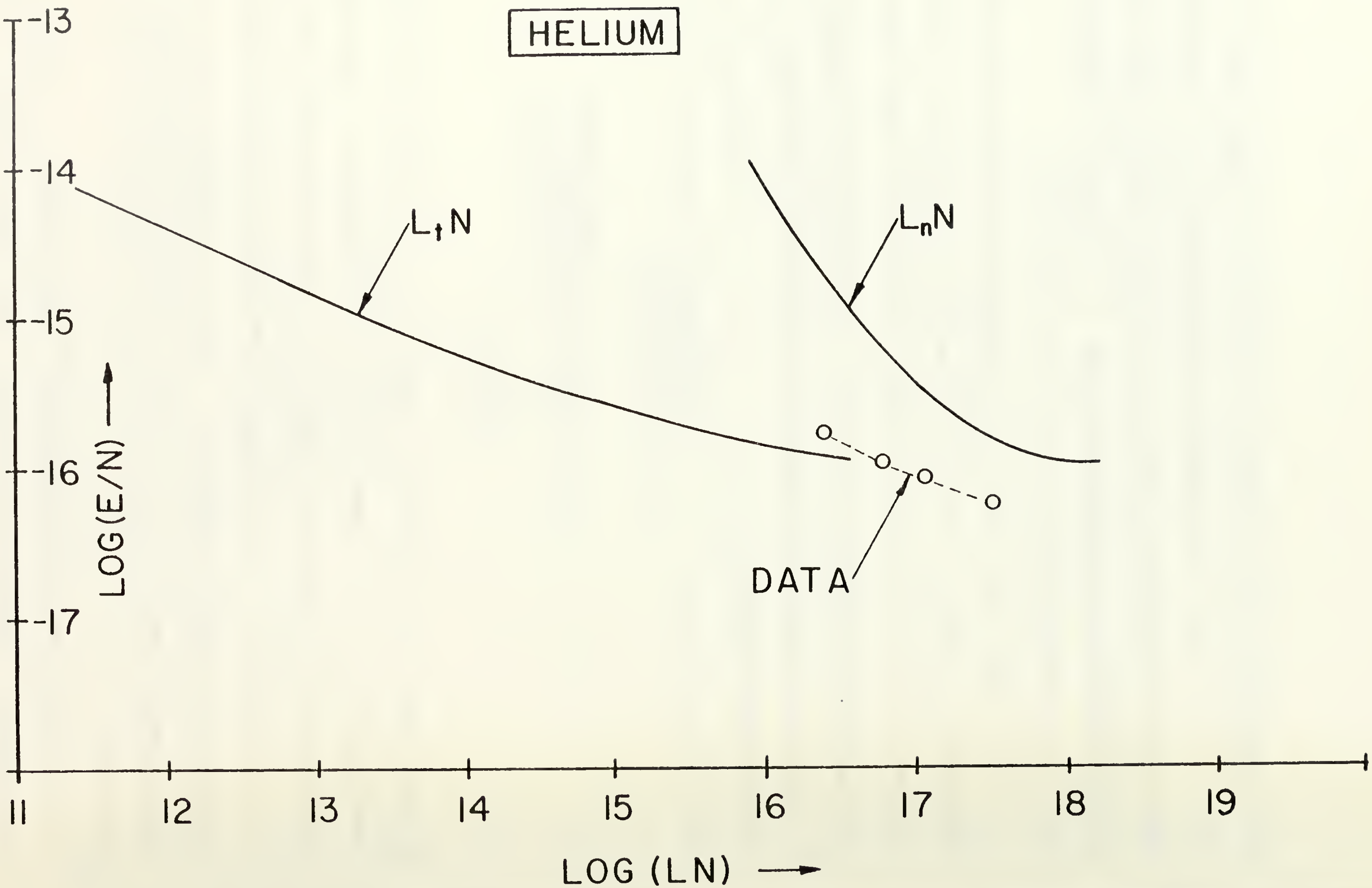
In the light of Eqn. (10), it is interesting to look at some actual values of L_n and L_T. Particularly extensive information for helium is reported in Ref. 16, and this is used in estimating the values shown in Fig. 2, which are limited to the range of applicability of the equations in Ref. 16. An appropriate form for the coefficients is obtained by introducing the gas density N_o,

$$L_n N_o = \sqrt{\frac{T_e (D_a/N_o)}{2 T_o (\nu_i/N_o)}} \tag{11}$$

$$L_T N_o = (E/N_o)^{-1} \sqrt{\frac{T_o K_o \alpha}{e(\mu_{eo} N_o)(\mu_i/N_o)}} \tag{12}$$

The above forms show "similarity" since LN is a function of E/N only. Examining Fig. 2, it is evident that $L_n N$ is not equal to $L_T N$, certainly not for large E/N, wheareas $L_T N$ follows closely data reported for helium in the literature[12,13] (the fact that we are looking at parallel plates and the data is for tubes is immaterial). Since in a discharge tube n and T share a common diameter, it is necessary to reconcile the discrepancy of the two curves.

It is possible, that the assumption[14] which brings T_e into Eqn. (1) and, therefore, into Eqn (11) is invalid. But, even if we were to eliminate


HELIUM
-13
-14
-15
-16
-17
LOG(E/N)
L$_t$N
L$_n$N
DATA
11
12
13
14
15
16
17
18
19
LOG (LN)

the factor T_e/T_o from the equation, the curves would diverge at the higher values of E/N. Constriction of the positive column is another possibility but in this case L_T would be expected to be greater than L_n for any given E/N and just the opposite seems to be true. Uncertainties in the estimate of α, the recombination coefficient, would affect the value of L_T but this estimate is the one that follows nicely the experimental values.

The method of reconciliation of the two lengths proposed here is based on the alternative that the equations previously written are incomplete and that, in particular, ν_i in Eqns. (1) and (7) should be replaced by a parameter (C_2) which makes up for the difference in the two curves. This matter is discussed in Appendix A of this report.

Consider now Eqn. (1) with constant coefficients. We proceed here to nondimensionalize it in a conventional way

$$- D_a n'' = \nu_i n - \alpha n^2 \tag{13}$$

$$n'(o) = 0 \quad \text{and} \quad n(\pm a) = 0$$

Let $n(o) = n_o$ and $\varepsilon = \alpha n_o/\nu_i$

Hence we get

$$- \hat{n}'' = \hat{n} - \varepsilon \hat{n}^2 \tag{14}$$

$$\hat{n}'(o) = 0 \quad \text{and} \quad \hat{n}(o) = 1$$

$$\hat{n}(\pm a/\sqrt{D_a/\nu_i}) = 0$$

Now the Shottky solution is one for which $\varepsilon = 0$, namely,

$$- \hat{n}'' = \hat{n} \tag{15}$$

$$\text{and} \qquad n = n_o \cos\left(\frac{\pi}{2} y/a\right) \tag{16}$$

It can be seen therefore that the characteristic length L should be

$$L = \sqrt{\frac{D_a}{\nu_i}} \doteq \frac{a}{\pi/2} \tag{17}$$

On the other hand, for the equilibrium limit $\varepsilon = 1$ and the equation

$$-\hat{n}'' = \hat{n} - \hat{n}^2 \tag{18}$$

is of the "Boundary Layer" type, i.e., it is not an eigenvalue problem, see Ref. 14 pp 149-51 for the equivalent problem with three-body recombination. The solution to Eqn. 18 in an implicit form is

$$y = \log \left\{ \frac{\sqrt{\frac{3}{2}} + (\hat{n} + \frac{1}{2})^{1/2}}{\sqrt{\frac{3}{2}} - (\hat{n} + \frac{1}{2})^{1/2}} \right\} - 1.317 \tag{19}$$

or explicitly

$$n = \frac{3}{2} n_o \left[\frac{1 - e^{-(y + 1.317)}}{1 + e^{-(y + 1.317)}} \right]^2 - \frac{n_o}{2} \tag{20}$$

which is indeed of the boundary-layer type. The intermediate cases, $0 < \varepsilon < 1$, are represented by elliptic functions, and this matter is discussed in Appendix B of this report.

IV. CONCLUSIONS

Some deficiencies of the Schottky solution have been re-examined with the intent of establishing a more useful solution for the description of the transition to the glow collapse. At least two major, recent attempts in the literature (Refs. 16 and 17) base the steady solution for the charge profile on the Schottky result and these attempts have not succeeded in presenting a complete formulation of the problem. The Technion approach[10] is to base the perturbation on a physically complete solution of the ambipolar charge profile. This profile must include the ingredients of constriction and of the elevation of the electron temperature.

As shown in Appendix A, the characteristic length scales for ambipolar diffusion and heat diffusion (conduction only) may be reconciled if ν_{io} in Eqn. (7) is replaced by C_2 where $C_2 > \nu_{io}$ and C_2 is a function of E/N (i.e., the dependance given by the difference of the two curves in Fig. 2). What appeared to be a bonus, namely, a insight into the mathematical description of striations, needs further study in order for it to make more physical sense. The eigenvalue nature of the diffusion equation is not charged by the recombination term except at the limit when $\varepsilon = 1.0$.

APPENDIX A - PROBLEM REFORMULATION

Equation (1) in the text presupposes steady flow along with $n = n(y)$. Let us relax these suppositions and go back to the species flux equations which may be written as

$$\frac{\partial n}{\partial t} + \nabla \cdot \left[n\left(- \mu_e \bar{E} - \frac{D_e}{n} \nabla n\right)\right] = \dot{n} \tag{A1}$$

$$\frac{\partial n}{\partial t} + \nabla \cdot \left[n\left(\mu_i \bar{E} - \frac{D_i}{n} \nabla n\right)\right] = \dot{n} \tag{A2}$$

Combining and eliminating the electric field we get the ambipolar diffusion equation in the usual manner which for constant coefficients is

$$\partial n/\partial t - D_a \nabla^2 n = \dot{n} \tag{A3}$$

$$\text{or} \quad \partial n/\partial t - D_a \left(\frac{\partial^2 n}{\partial x^2} + \frac{\partial^2 n}{\partial y^2}\right) = \nu_i n - \alpha n^2 \tag{A4}$$

The above assumes two-body recombination as well. Now, using Eqn. (A3) to eliminate $\nabla^2 n$ from Eqn. (A2) yields

$$\frac{\partial n}{\partial t} + \frac{\mu_i \nabla \cdot n\bar{E}}{1 - D_i/D_a} = \dot{n} \tag{A5}$$

But since $\nabla \cdot \bar{E} = 0$ in the ambipolar region

$$\frac{\partial n}{\partial t} + \frac{\mu_i}{1 - D_i/D_a} \bar{E} \cdot \nabla n = \dot{n} \tag{A6}$$

$$\text{or} \quad \partial n/\partial t + \left(\frac{\mu_i}{1 - D_i/D_a}\right)\left(E_L \frac{\partial n}{\partial x} + E_y \frac{\partial n}{\partial y}\right) = \nu_i n - \alpha n^2 \tag{A7}$$

Let us work with Eqns. (A4) and (A7) in linear form, i.e., neglecting recombination. If

$$n(x,y,t) = f(y)\ g(x,t) \tag{A8}$$

Then, substituting into Eqn. (A4)

$$f\ g_t - D_a\ (f g_{xx} + g f'') = \nu_i f g - \ldots \tag{A9}$$

where $\quad f'' = \dfrac{d^2 f}{dy^2} \qquad g_{xx} = \dfrac{\partial^2 g}{\partial x^2}$ and $g_t = \partial g/\partial t$

Dividing by fg both sides of (A9)

$$1/g\ (g_t - D_a\ g_{xx}) - \frac{D_a}{f} f'' = \nu_i \tag{A10}$$

At a given E/N, ν_i is constant so we may presume that

$$1/g\ (g_t - D_a\ g_{xx}) = \text{constant (or } - C_1) \tag{A11}$$

and

$$- \frac{D_a}{f} f'' = \text{constant (or } C_2) \tag{A12}$$

with

$$- C_1 + C_2 = \nu_i \tag{A13}$$

The constants C_1 and C_2 are positive and defined so that $C_2 \geq \nu_i$ as required by Eqn. (A12), which now replaces Eqn. (6) since

$$- D_a\ f'' = f\ C_2 \tag{A14}$$

and $\quad L_f \equiv \sqrt{D_a/C_2}$

Equation (A14) is the new Schottky equation. The constant C_1 appears in the companion partial differential equation

$$g_t - D_a g_{xx} + C_1 g = 0 \tag{A15}$$

Now look at Eq. (A7). At the centerline $\partial n/\partial y = 0$ and $f(o) = f_o$ so that

$$\text{at } y = 0 \qquad \frac{\partial n}{\partial t} + \left(\frac{\mu_i}{1 - D_i/D_a}\right) E_L \frac{\partial n}{\partial x} = \nu_i n - \ldots \tag{A16}$$

Introducing $n = fg$ we obtain

$$g_t f_o + \left(\frac{\mu_i}{1 - D_i/D_a}\right) E_L g_x f_o = \nu_i f_o g \tag{A17}$$

We can cancel f_o in the linear equation above and obtain an equation in which no function of y appears

$$g_t + \left(\frac{\mu_i}{1 - D_i/D_a}\right) E_L g_x = \nu_i g \tag{A18}$$

At this point it is worthwhile to eliminate g_t between Eqns. (A18) and (A15) and obtain

$$D_a g_{xx} - C_1 g = \nu_i g - \left(\frac{\mu_i}{1 - D_i/D_a}\right) E_L g_x$$

$$D_a g_{xx} + \frac{\mu_i E_L}{(1 - D_i/D_a)} g_x = (\nu_i + C_1) g$$

$$\text{or} \qquad g_{xx} + \left(\frac{\mu_i E_L}{D_a - D_i}\right) g_x = \frac{C_2}{D_a} g \tag{A19}$$

Equation (A19) can be solved with

$$g = T(t)\ e^{\alpha x} \tag{A20}$$

where α is found from

$$\alpha = \frac{-\xi \pm \sqrt{\xi^2 + 4C_2/D_a}}{2} \qquad \xi = \frac{\mu_i E_L}{D_a - D_i} \tag{A21}$$

Note that $D_i < D_a$ and that ξ is anyway a positive parameter. Equation (A21) indicates that the roots are real and there is no periodic behavior in x -- that striations are not to be found with this description.

In order for n to oscillate in x, we must have a situation in which new constants $C_1{}^*$ and $C_2{}^*$ are introduced such that

$$C_1{}^* - C_2{}^* = \nu_i \tag{A22}$$

$$g_t - D_a\, g_{xx} = C_1{}^*\, g \tag{A23}$$

and

$$-D_a\, f'' = -C_2{}^*\, f \tag{A24}$$

Here, Eqn. (A24) will result in (non-periodic) hyperbolic functions in y and Eqn. (A24) in oscillatory functions in x provided that

$$\frac{4C_2{}^*}{D_a} > \left(\frac{\mu_i E_L}{D_a - D_i}\right)^2 \tag{A25}$$

As before we would like to have

$$C_1{}^* > \nu_i \quad (C_2{}^* = C_1{}^* - \nu_i) \tag{A26}$$

so as to match L_n to L_T.

The solution to Eqn. (A24) does not represent the usual diffusion controlled situation because in order to meet the symmetry condition at the centerline we must give up the physically sensible boundary conditions at the wall. Perhaps variable coefficients and the inclusion of recombination can remedy this problem with the boundary conditions. Or perhaps we should be willing to give up axial symmetry and consider some sort of contraction asymmetry which would likely have to include some reconbination because of the higher pressures. This, however shall not be pursued in this report.

APPENDIX B - INFLUENCE OF TWO-BODY RECOMBINATION OF SCHOTTKY SOLUTION

The diffusion equation given in the main text (Eqn. 14) is not suitable for elliptic funciton representation, therefore, a different form is developed below.

The diffusion equation with constant coefficients is (without consideration of whether C_2 shall be used)

$$- D_a \frac{d^2 n}{dy^2} = \nu_i n - \alpha n^2 \tag{B1}$$

Defining a stationary state $\nu_i n^* - \alpha n^{*2} = 0$, or

$$n^* = \nu_i / \alpha \tag{b2}$$

We can nondimensionalize the equation with

$$\hat{n} = n/n^* \quad \text{and} \quad \hat{y} = y/L_n$$

Obtaining

$$- \frac{d^2 \hat{n}}{d\hat{y}^2} = \hat{n} - \hat{n}^2 \tag{B3}$$

$$n'(o) = 0 \quad \text{and} \quad n = \beta \ , \quad 0 \leq \beta \leq 1$$

$$n(a/L_n) = 0$$

It is clear that the boundary condition at the centerline make up for the disappearance of ε in Eqn. (B3). Now Eqn. (B3) can be integrated once to yield

$$\frac{d\hat{n}}{d\hat{y}} = \sqrt{\left(\frac{2}{3}\hat{n}^3 - \hat{n}^2\right) - \left(\frac{2}{3}\beta^3 - \beta^2\right)} \tag{B4}$$

or $$\sqrt{\frac{3}{2}} \int_{\hat{n}}^{\beta} \frac{d\hat{n}}{\sqrt{n^3 - \frac{3}{2} n^2 - \beta^3 + \frac{3}{2} \beta^2}} = \int_{\hat{y}}^{\infty} d\hat{y} \tag{B5}$$

The left-hand-side of the above equation can be represented in terms of elliptic functions. Let

$$(\hat{n}^3 - \frac{3}{2} \hat{n}^2 - \beta^2 + \frac{3}{2} \beta^2) = (n - a)(n - b)(n - c) \tag{B6}$$

therefore

$$\left.\begin{aligned} a &= \frac{\left(\frac{3}{2} - \beta\right) + \sqrt{\left(\frac{3}{2} - \beta\right)\left(\frac{3}{2} + 3\beta\right)}}{2} \\ b &= \beta \qquad (0 \leq \beta \leq 1) \\ c &= \frac{\left(\frac{3}{2} - \beta\right) - \sqrt{\left(\frac{3}{2} - \beta\right)\left(\frac{3}{2} + 3\beta\right)}}{2} \end{aligned}\right\} \tag{B7}$$

where $a \geq b \geq \hat{n} \geq c$

With information on a, b, and c we can enter any table for elliptic functions (see for example Ref. 18 or 19). Using Byrd and Friedman notation, formula 234.00, we have

$$\int_{\hat{n}}^{b} \frac{d\hat{n}}{\sqrt{(a - \hat{n})(b - \hat{n})(\hat{n} - c)}} = gF\,(\varphi,k) \tag{B8}$$

where

$$g = \frac{2}{\sqrt{a - c}}, \qquad k^2 = \frac{b - c}{a - c}$$

and $$\varphi = \sin^{-1}\sqrt{\frac{(a - c)(b - \hat{n})}{(b - c)(a - \hat{n})}}$$

$F(\varphi,k)$ is tabulated. Note that for $\beta = 1$, we get a double root $(a = b)$ and the equation is not an elliptic integral (though its solution can still be found from the Tables). When $\beta = 1$ we get the boundary-layer behavior described by Eqn. (19) or (20) in the main text.

Except for $\beta = 1$, Eqn. B3 represents an eigenvalue problem. In order to check this, we solve for the value of the integral at the wall where $\hat{n} = 0$. Thus

$$\varphi = \sin^{-1}\sqrt{\frac{(a - c)b}{(b - c)a}}$$

For very small values of β, we obtain the cosine solution, namely,

$$\lambda = (\pi/2)^2 \quad \text{where} \quad L_n = a/\lambda^{1/2}$$

As β increases, we obtain progressively larger values until at $\beta = 1$ we get $\lambda = \infty$). Figure B1 shows this behavior for the eigenvalue. The results shown in Fig. B1 proved identical to results from a computer program at the Technion (for the combined diffusion and heat dissipation).

Increasingly higher degrees of recombination flatten out the density profile at the center and increase its slope at the walls. The radial electric field is thus enhanced. The variable coefficient solution is not anticipated to change the above results in any major way.

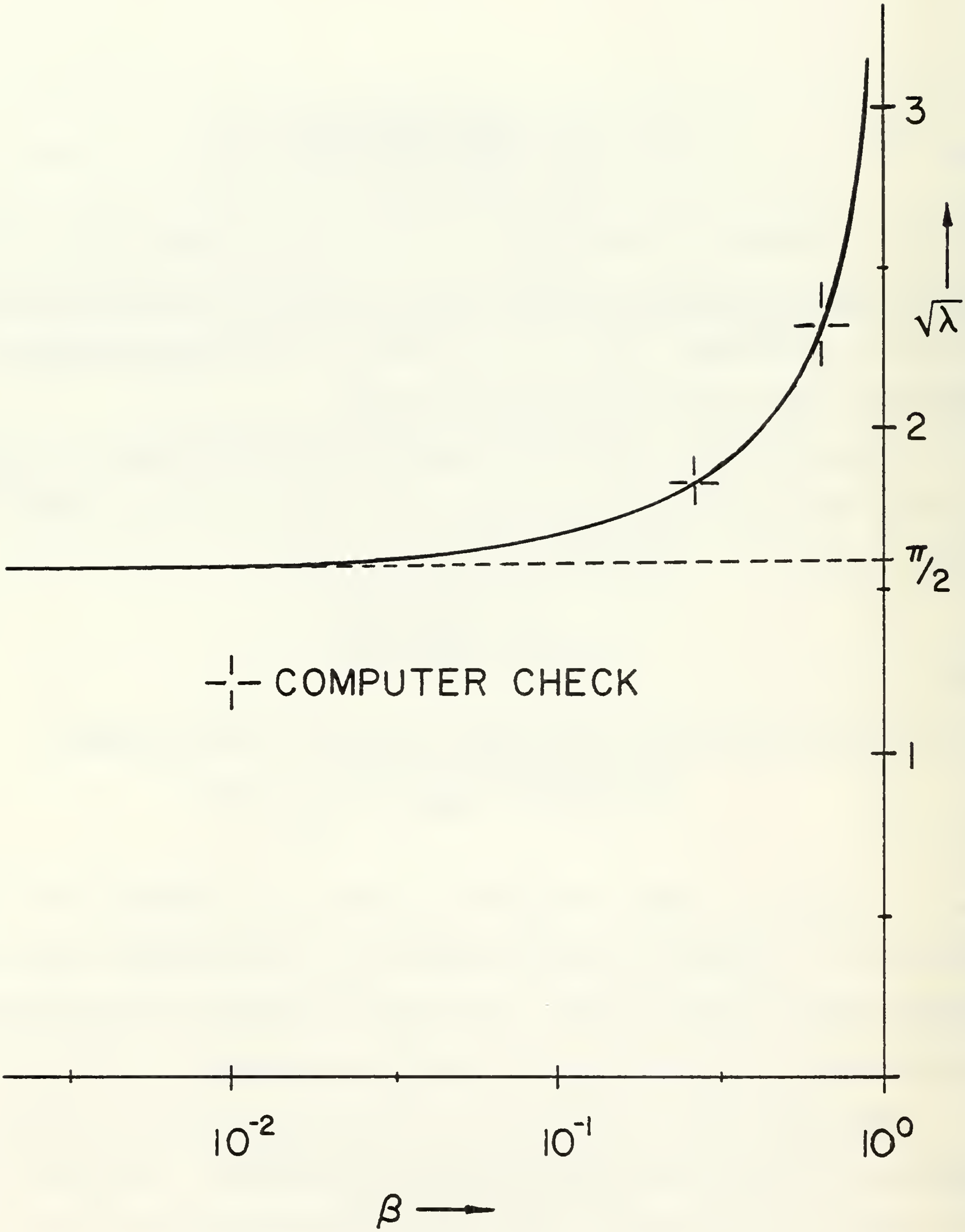


Figure B1. Eigenvalue for Increasing Degrees of Recombination

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